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REPORT UPON THE APPORTIONMENT OF REPRESENTATIVES*

In accordance with the request of the Senate Committee on the Census, transmitted by your letter of February 18, 1921, our committee has examined the proposed methods of apportioning representatives among the several states. We have had before us documents and correspondence furnished by the Director of the Census, by Dr. J. A. Hill of the Bureau of the Census, and by Professors W. F. Willcox and F. W. Owens of Cornell University and E. V. Huntington of Harvard University. Professor Willcox is a member of our committee, but, because he devised one of the methods considered, he does not join in this report. Professors Huntington and Owens, upon our invitation, appeared before the committee and explained the methods they had devised. We have also sought the counsel of Professor Irving Fisher of Yale University. Our conclusions may be summarized as follows:

1. The "method of equal proportions" leads to an apportionment in which the ratios between the representation and the population of the several states are as nearly alike as is possible. It thus complies with the conditions imposed by a literal interpretation of the requirements of the Constitution.

2. The "method of major fractions" has back of it the weight of precedent. Logically, however, it can be supported only by holding that the Constitution requires, not that the ratios between the representation and the population of the several states shall be equal, as nearly as is possible, but that the representation accorded to individuals or to equal groups of individuals in the population (that is, their "shares" in their respective representatives) shall be as nearly uniform as is possible, irrespective of their places of residence.

3. It is not clear that the special interpretation of the Constitution which alone is consistent with the use of the "method of major fractions" is to be preferred to other possible special interpretations which lead to other methods of apportionment. We conclude, therefore, that the "method of equal proportions," consistent as it is with the literal meaning of the words of the Constitution, is logically superior to the "method of major fractions."

In the sections which follow we explain the considerations which have led us to these conclusions.

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* This report was made by the Joint Committee of the American Statistical Association and the American Economic Association to Advise the Director of the Census in the form of a letter to United States Senator Howard Sutherland, chairman of the Senate Committee on the Census.

I. THE ALTERNATIVE METHODS

The two methods of apportioning representatives with respect to which the judgment of the committee is sought are, we understand, first, the "method of major fractions," devised by Professor W. F. Willcox in 1910 and used in the apportionment of 1911,* and, second, the "method of equal proportions," devised by Professor E. V. Huntington in December, 1920, and brought to the attention of Congress through a letter to Chairman Siegel of the House Committee on the Census.† To the "method of major fractions" there may be joined a method devised by Professor F. W. Owens in February, 1921, and named by him the "method of least errors." Professor Owens' method, it happens, although independently conceived and resting on somewhat different premises, leads to precisely the same apportionment as the "method of major fractions." It may be taken, therefore, as adding strength to the claims of that method. Moreover, Professor Huntington's "method of equal proportions" is in effect a modified form of the "method of alternate ratios," devised by Dr. J. A. Hill of the Bureau of the Census and recommended by Dr. E. Dana Durand, then Director of the Census, for adoption in 1911.‡ In discussing the "method of major fractions" we have taken into account, therefore, the suggestions of Professor Owens as well as those of Professor Willcox, and similarly we have joined Dr. Hill's suggestions to those of Professor Huntington in our consideration of the "method of equal proportions."

II. FACTORS THAT INFLUENCE A CHOICE

Judged by purely technical criteria, the method of equal proportions is superior to the method of major fractions. By "purely technical criteria" we do not mean matters of mathematical detail. Both methods are, in such respects, mathematically sound. The difference between them is one of logic rather than mathematics, of premises rather than of methods of deduction. Both methods are logically defensible. The choice between them is not a choice between the right and a wrong method. The question is a more difficult one: which of two defensible methods is the better?

We say that we base our judgment on "purely technical criteria" in order to make it clear that we have left out of account, as beyond our province and our competence, certain matters which Congress perhaps may deem important. In particular we have given no weight either to precedent and tradition or to subtleties of interpretation, such as might find in the language of the Constitution, or in the circumstances of its history, an indication that some one particular significance or meaning should be attached to apportionment ratios.

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* Cf. Report of the House Committee on the Census, 1911, 82d Congress, 1st Session, House Report No. 12, pp. 1–42.

† Congressional Record, January 19, 1921, p. 1791.

‡ Report of the House Committee on the Census, 1911, pp. 43–108.

If any weight is to be given to precedent, it will count for the method of major fractions. Professor Willcox, who devised the method used in the apportionment of 1910, has explained that his aim was "simply to provide a set of tables which would conform to the desire and purpose of Congress," as expressed in previous reports, debates, and votes upon apportionment. In the past Congress has, in fact, indicated its belief that if a state's population exceeds any exact multiple of the population of the average congressional district by a "major fraction," the major fraction gives the state a prima facie claim to another representative. The method used in the apportionments of 1840 and 1910, and now called the method of major fractions, satisfies the requirements a recognition of such claims makes as closely as is practicable.

What weight should be attached to precedent is, of course, for Congress to determine. While, on the one hand, it might be urged that if any other method is to displace the method of major fractions the other method must have a very considerable margin of superiority, it might possibly be maintained, on the other hand, that in view of the inherent complexities of the problem and the likelihood that all of its different aspects have not been presented to Congress in the past, it would be wrong to consider the case as in any degree closed. At any rate, we venture to suggest, there is one factor whose importance has increased and is likely to increase. In the past the increased representation the more rapidly growing states have gained has seldom been at the direct expense of other states. With the interests of each state thus, in a measure, safeguarded, some of the mathematical niceties of exact apportionment might be disregarded. But once the size of the House of Representatives is definitely restricted, the problem of apportionment takes on new importance. It is no longer merely a question of what states shall gain representatives. It includes also the more awkward question of what states shall lose representatives. It is possible that under such conditions Congress may hold that it is less important that the method of apportionment be based on precedent than that it should rest on the best possible logical and mathematical foundation, so as to be immune, so far as may be, from successful criticism and attack.

III. THE CONSTITUTIONAL REQUIREMENT

We do not understand that the constitutionality of either of the proposed methods is now in question, nor do we assume that the constitutionality of either of them is likely to be challenged. Both methods comply with the requirements of the Constitution in that each leads to an apportionment of representatives "among the several states according to their respective numbers." The problem merely is, which method leads to a more nearly *exact* apportionment?

The words of the Constitution are: "Representatives shall be apportioned among the several states according to their respective numbers, counting the whole number of persons in each state, excluding Indians not taxed." (Fourteenth Amendment, Sec. 2.) As we have said, we do not conceive our task to be that of constitutional interpretation. Nevertheless it is impossible to come to any conclusion respecting the relative merits of the competing methods of apportionment without careful consideration of their relation to the requirements of the Constitution. For this purpose we have taken the words of the Constitution at their face value. We have tried to avoid reading into them any implications not warranted by a rigidly literal interpretation.

For example, it has been held that these words imply that a group of individuals resident in any part of the United States is entitled to the same amount or share of representation in Congress as any other group of individuals of equal numbers resident in any other part of the United States. Or it might be held that it is implied that the population of the different congressional districts should be as nearly equal as possible. These two implications, seemingly somewhat alike, lead logically to two very different methods of apportionment. Congress might draw inferences like these from the stipulations of the Constitution. Our committee has not ventured to go that far. For a similar reason we have given no weight to the suggestion that the equal representation of the states in the Senate indicates that preference should be given to that method of apportioning representatives in the House which is more favorable to the larger states. That is clearly a matter of congressional policy, outside the field of inquiry assigned to our committee.

Confining ourselves, then, to a literal reading of the constitutional clause, we find in it two guiding principles that have an important bearing upon the problem in hand.

First, representatives are to be "apportioned among the several states." We have not thought it advisable for us to go beyond this clear stipulation by treating the problem as one of adjusting the equities of individual citizens, of groups of individuals, or of their representatives. As we have suggested, the restatement of the problem in some such manner might point to a different solution from the one we favor. In its original form (Constitution, Article I, Sec. 2), the clause in question, covering as it did the apportionment of both representatives and direct taxes, was born of an effort to adjust the conflicting interests of the different states. We have not assumed that its purpose and meaning have changed.

Second, representatives are to be apportioned among the states *according to their respective numbers.* Here we have a plain stipulation that the allocation of representatives among the states is to be determined on the basis of *proportions* or *ratios.* The ratio between the representation assigned any one state and the representation assigned any other state is to accord with the ratio between the population of the one and the population of the other. But there is no stipulation with respect to the precise way in which such proportions are to be expressed or stated.

IV. THE MEASUREMENT OF THE EQUALITY OF RATIOS

No method of apportionment, of course, can assign representatives to the several states in *exact* proportion to their respective numbers. The problem then is, in Daniel Webster's phrase, to reach a result that shall be "as near as may be" to an exact apportionment. The principal difficulty occurs just at this point. How is "nearness" to an exact apportionment to be measured? How is the difference between two states with respect to the relation between their representation and their numbers to be determined? These questions are included, in part at least, in a larger problem—how is the "nearness" of any two numbers to be determined?

The "nearness" of two numbers may be measured either *absolutely* or *relatively.* For example, which are nearer, the numbers 3 and 5, or the numbers 45 and 50? The absolute difference between 3 and 5 is only 2, while between 45 and 50 it is 5, so that by this test 3 and 5 are "nearer" than 45 and 50. But while 3 is 60 per cent of 5, 45 is 90 per cent of 50, so that *relatively* 45 and 50 are "nearer" than 3 and 5. Suppose that state A is given 20 representatives, its exact quota being 20.6, while state B is given 2 representatives, its exact quota being 2.4. Which state is nearer to its exact quota? Absolute nearness is closer in the case of state B, since 0.4 is smaller than 0.6; but relative nearness is closer in the case of state A, since state A has 97 per cent of its exact quota, while state B has only 83 per cent. Expressed in this manner, the measurement of "nearness" by absolute differences leads to the method of major fractions, while its measurement in relative or percentage terms leads to the method of equal proportions.

For the general problem of the measurement of the "nearness" of two numbers, there is no single exclusive solution. The appropriate method must be determined by the nature of the concrete problem in hand. For example, temperatures of 5 degrees and 10 degrees above zero are "nearer" than are temperatures of 90 degrees and 100 degrees. But a firm with assets of $50,000 and liabilities of $100,000 is further from solvency than a firm with assets of $900,000 and liabilities of $1,000,000.

Although the problem of the best method of measuring the "nearness" of *numbers* is thus, as a general problem, ambiguous, such is not the case with respect to the "nearness" of *ratios* or proportions. A ratio between numbers is not itself a quantity, yet there is no way of measuring the absolute difference between ratios, except by putting them into the form of fractions and treating them as numbers or quantities. In order that a ratio may be treated as a fractional quantity, certain assumptions must be made which may not, in a particular case, be warranted. But ratios may properly be compared, *as ratios*, with respect to their *relative* "nearness." Thus to say that the ratio between 2 and 4 is 83⅓ per cent of the ratio between 3 and 5, is merely to say that the first ratio is related to the second ratio as the ratio between 83⅓ and 100 is related to the ratio between 100 and 100.

The problem in hand, or rather so much of that problem as we take to be within the field of our inquiry, is to determine which of the methods under review will give an apportionment of representatives that will be, "as near as may be," in accordance with the populations of the several states. This is a matter of making ratios or proportions as nearly alike as possible. For the reasons we have outlined the alikeness or nearness of ratios is best measured in relative terms. This is one ground on which technical superiority must be assigned to the method of equal proportions.

V. DIFFERENT WAYS OF EXPRESSING RATIOS

When apportionment ratios are expressed as fractions, further difficulties appear. Such fractions may take several forms. For example, their numerators may express the number of representatives to be assigned the several states, and the denominators their populations. Or population may form the numerator and the number of representatives the denominator. Or, again, the representation of any two states may be compared by means of a pair of fractions. One fraction has for its numerator the population of one state and for its denominator the population of the other. The other fraction has for its numerator the number of representatives assigned the first state, and for its denominator the number of representatives assigned the other.* Which among these different sets of fractions is to be selected?

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* Denoting the number of representatives assigned to states A and B by a and b, and their respective populations by A and B, there are the following possible classes of fractions: (1) a/A, b/B, etc.; (2) A/a, B/b, etc.; (3) A/B, a/b, etc.; (4) B/A, b/a, etc.

If the method of equal proportions is used, the choice is a matter of indifference. Any or all of the possible forms of fractions will lead to the same apportionment. That is, the apportionment which will bring one set of fractions "as near as may be" to equality, will also minimize the differences in the other sets of fractions. This is because when the "nearness" of ratios is measured on a relative scale, the results do not depend upon the particular form of the fractions by means of which the ratios are expressed.

When, however, the "nearness" of fractions is measured by their absolute differences, so that the fractions are treated as quantities rather than as merely expressing ratios, the form of the fraction used in the comparison becomes a matter of importance. The measurement of "nearness" by means of absolute differences does not of itself lead to the method of major fractions. That method requires the further assumption that the significant fractions are those in which the numerators denote representatives and the denominators population. Inverted, so that their numerators denote population, and with their "nearness" measured by their absolute differences, these fractions lead to a very different scheme of apportionment, which, on mathematical (though not on historical) grounds has an equally valid claim to the name, method of major fractions. The method of equal proportions, it is interesting to note, gives results that are intermediate between those given by these two ways of measuring "nearness" by absolute differences. It is somewhat more favorable to the small states than is the method of major fractions and somewhat more favorable to the large states than that method would be if it made use of the other form of fractions.

On purely mathematical grounds there is no reason to prefer one of these two forms of fractions to the other. The two are equally valid as expressions of apportionment ratios. It thus appears that unless other than mathematical reasons can be found for preferring one form of fraction, the method of major fractions encounters the formidable objection that it leads to results that are self-contradictory.

If preference is to be given to one form of fraction, it can only be because different *meanings* are attributed to the different forms. Now as long as these fractions are construed merely as expressing ratios, there can be no difference in their meanings. To state that the ratio of representatives to population is as 1 to 240,000 is to state that the ratio of population to representatives is as 240,000 to 1. A familiar example may make the matter clear. The speed of a railway train or of an aeroplane is stated in terms of miles per hour; the speed of a runner in terms of minutes per mile. The meaning is the same in both cases: the ratio between the two magnitudes: distance traversed and time elapsed. Science, it is true, has adopted the convention of expressing the time-distance ratio as a fraction in which distance is the numerator and time the denominator. But this is merely a matter of convenience. In inverted form the ratio, *as a ratio*, would have the same *meaning*.

If distinctive meanings, therefore, are to be attached to the different forms of fractions which express apportionment ratios, they must be construed to be something more than mere ratios. This construction may be accomplished by using common denominators or by reducing the denominators to unity, so that the numerators become the quotients of the original numerators divided by the original denominators. To the numerators, taken by themselves, concrete meanings can be attributed. Thus, the number of representatives assigned to a state divided by a number representing its population gives a number which may be interpreted as the "average representation accorded to an individual in the state's population." Similarly, the quotient obtained by dividing the state's population by a number corresponding to the representatives assigned it, may be construed as "the average size of a representative's constituency," or "the average size of a congressional district within the state."

So far as mathematical considerations are concerned, the processes by which one meaning or the other is imputed to apportionment ratios are wholly arbitrary. If the method of major fractions is to be supported, it must be by holding that what the Constitution requires is, after all, not that the ratios of representatives and population shall be equal "as near as may be" for the several states, or even that the average size of congressional districts shall be as equal as possible as among the several states, but that the differences in "the representation accorded an individual" shall be minimized. The absence of any clear reasons (save those of precedent) for emphasizing the "representation accorded an individual" rather than "the size of an average congressional district," or either of them rather than that simple proportioning of representatives according to population which the constitutional clause, taken at its face value, seems to require, establishes the second ground on which the method of equal proportions is to be preferred to the method of major fractions.

VI. THE USE OF SQUARES IN THE METHOD OF LEAST ERRORS

Professor Owens has shown that the apportionment which will make the sum of the squares of the errors (*i. e.*, the differences between the actual share a citizen of a state has in his representative and the exact share to which he is "entitled") a minimum will be the same apportionment as would be reached by the method of major fractions. This result affords additional evidence that results which the method of major fractions gives are logical deductions from the premises of that method, and possibly makes the logical premises of that method somewhat clearer. But this use of the method of least squares does not strengthen the claims of the method of major fractions as against the method of equal proportions. The "errors" whose squares Professor Owens seeks to minimize correspond to those "absolute differences" which are less appropriate measures of the "nearness" of apportionment ratios than are the relative differences utilized in the method of equal proportions. Moreover, the method of least squares is applicable to any of the forms of fractions which may be used to express apportionment ratios, and thus leads to self-contradictory results. In short, as used by Professor Owens it not only presupposes the validity of absolute differences, as a measure of "nearness," but it also requires, as Professor Owens himself suggests, that only one of the possible types of fractional expression of the apportionment ratios shall be utilized.

VII. SUMMARY OF CONTROLLING CONSIDERATIONS

1. It is clear that the Constitution requires that the allocation of representatives among the several states shall be proportionate to the distribution of population. It is not equally clear that there is anything in the constitutional requirement which suggests that one of the forms in which such apportionment ratios or proportions may be expressed should be preferred to another.
2. The method of major fractions utilizes only one of several ways of expressing apportionment ratios. The method of equal proportions utilizes all of these ways without inconsistency. The latter method, therefore, has a broader basis.
3. There is no mathematical or logical ground for preferring the one form of expression of the apportionment ratio used in the method of major fractions to other forms of expression. These other forms lead, when similar processses of computation are employed, to different and therefore inconsistent results.
4. The method of major fractions logically implies preference for a special meaning which may be attached to one of the forms in which apportionment ratios may be expressed. To attach to ratios meanings which vary with the forms in which the ratios are expressed is to interpret them as something else than ratios.
5. In the method of major fractions the "nearness" of the ratios

of representatives and population for the several states is measured by absolute differences. The method of equal proportions utilizes relative differences. The relative scale is to be preferred.

Respectfully submitted
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A NATIONAL INDEX OF RETAIL TRADE

The Federal Reserve system is at present receiving monthly reports from about 350 department stores, located in all sections of the United States. The statistics obtained from these stores are as follows: percentage change of net sales from sales in corresponding month of previous year, percentage change of stocks from corresponding month of previous year, percentage change of stocks from previous month, and ratio of outstanding orders at close of month to total purchases in previous year.

Indices for these items have been published for each of the twelve Federal Reserve districts since August, 1920, but no national index was made previous to August, 1921. At that time a national index was constructed by combining weighted percentages for the several districts. These weights were secured by finding the percentage distribution of population and of banking resources, respectively, in 1920, among the Federal Reserve districts. A weight is obtained for the retail trade of each district in its relation to the retail trade of the nation by multiplying the population percentage of the district by two and adding the district's percentage of total banking resources.

The monthly comparisons of changes in retail sales and stocks since August, 1920, for the United States are shown in the following table:

TABLE I

CHANGES IN VOLUME OF SALES AND OF STOCKS IN COMPARISON WITH CORRESPONDING MONTH IN PREVIOUS YEAR

	Sales Per cent	Stocks Per cent
1920		
August	+21.2	+37.0
September	+13.6	+28.2
October	+10.0	+26.4
November	+13.4	+16.7
December	+ 3.1	+ 6.2
1921		
January	− 4.7	− 9.4
February	+ 0.8	−14.4
March	− 3.0	−17.9
April	− 4.2	−18.9
May	− 8.6	−17.7
June	−10.7	−17.4
July	−15.1	−19.1
August	−11.9	−19.0